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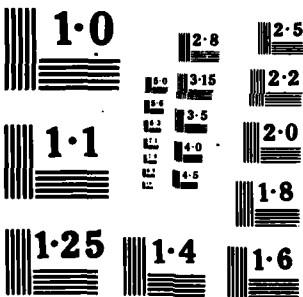
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AERO PROPULSION REPORT 166

A THEORETICAL STUDY OF TWO STAGE  
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by

A. M. ABDEL-FATTAH

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**SUMMARY**

The results of theoretical assessment of two stage thrust augmenting ejectors are presented and compared with those of single stage ejectors. The mixing ducts were of constant cross sectional area, the flows at the inlet and exit planes of each stage were assumed to be uniform, and friction effects were ignored.

It was found that staging the ejector increases thrust augmentation at all primary jet stagnation pressures, but is more effective in the low pressure range and with high ejector area ratios for any gas combination. With a Hot Rocket Gas-Air Combination, the benefit of staging is much less than with an unheated Air-Air combination, and does not appear to be of practical use.



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## NOMENCLATURE

<i>A</i>	Duct or flow cross sectional area.
<i>C<sub>p</sub></i>	Specific heat at constant pressure.
<i>F<sub>n</sub></i>	Nozzle thrust.
<i>F<sub>e</sub></i>	Ejector thrust.
<i>H.R.G.</i>	Hot Rocket Gas.
<i>M</i>	Mach number.
<i>ṁ</i>	Mass flow rate.
<i>P</i>	Static pressure
<i>P<sub>0</sub></i>	Total pressure.
<i>R</i>	Gas constant.
<i>R</i>	Universal gas constant.
<i>T</i>	Static temperature.
<i>T<sub>0</sub></i>	Total temperature.
<i>V</i>	Flow velocity.
<i>W</i>	Molecular weight.
<i>γ</i>	Ratio of specific heats.
<i>μ<sub>1</sub></i>	Secondary mass flow ratio = $\frac{\dot{m}_1}{\dot{m}_p}$ .
<i>μ<sub>2</sub></i>	Tertiary mass flow ratio = $\frac{\dot{m}_2}{\dot{m}_p}$
<i>τ</i>	Thrust augmentation ratio = $\frac{F_n + F_e}{F_{n_a}}$ .
<i>ϕ</i>	A function defined in text.
<i>ψ</i>	Effectiveness of staging = $\frac{\tau_{2s}}{\tau_{1s}}$
<i>ρ</i>	Density.
<i>Γ</i>	Improvement of mass augmentation by staging = $\frac{(1 + \mu_1 + \mu_2)_{2s}}{(1 + \mu_1)_{1s}}$
<i>Subscripts</i>	
1, 2, 3, 4	Relating to stations 1, 2, 3 and 4 in figure 2.
<i>a</i>	Relating to ambient conditions.
<i>p</i>	Relating to primary jet flow.
<i>1s</i>	Single stage ejector
<i>2s</i>	Two stage ejector
*	Relating to nozzle throat.

## 1. INTRODUCTION

An ejector is a device in which part of the energy in a relatively high velocity primary jet from a nozzle at the inlet is used to entrain a stream of low energy secondary fluid (e.g. ambient air) within a confining duct, as shown in figure 1. The degree of entrainment depends mainly on the effectiveness of the kinetic and thermal mixing between the two streams in the duct. The momentum flux of the mixed flow emerging from the mixing duct is generally large compared with that in the primary jet, the increased mass flow outweighing the reduction in primary fluid mean velocity. The reduction in jet velocity improves the propulsive efficiency, especially for a jet issuing from a vehicle which is stationary or (say) hovering at low forward speeds.

The present study forms part of a broader investigation of the use of ejectors for improving the static thrust of rocket motors. In terms of primary jet stagnation pressure and temperature, this application is well outside the experience of most previous workers (Viets 1975, Quin 1976). Recent work with jets of this nature (Fisher 1980, Fisher and Irvine 1981) was confined to single stage ejectors. Although multi staging the ejector has been found to increase the availability of thrust experimentally (Morrison 1942), and analitically (Nagaraja *et al.* 1973), no further work to the best knowledge of the author has taken place. These two references were cited by Viets (1975), but were not available to the present author. It is the purpose of the present work to evaluate theoretically the availability of thrust with two stage ejectors, with an emphasis on relatively high primary jet stagnation pressure ratios, such as those used in rocket motors.

## 2. THEORETICAL CONSIDERATIONS

One dimensional compressible flow theory is adopted with the ejector configuration shown in figure 2.

The primary jet flows through an area  $A_p$  and entrains the secondary air through the annular area  $A_1$ . The two streams mix in the first duct of cross sectional area  $A_4$ . This mixed flow entrains the tertiary air through the annular area  $A_2$  and mixes with it in the second duct of cross sectional area  $A_3$  before exhausting to the atmosphere.

The following are assumed.

- a. Both mixing ducts are of constant cross sectional area.

$$\begin{aligned} A_3 &= A_4 + A_2 \\ &= A_p + A_1 + A_2 \end{aligned}$$

- b. Static pressure across the inlet and exit planes of each duct is uniform.

$$P_p = P_1, P_4 = P_2, P_3 = P_b$$

- c. The flow distribution in each stream at the inlet and exit of each duct is uniform.

- d. Skin friction is neglected.

- e. The gases are compressible and satisfy the perfect gas law throughout the mixing process, with specific heats independent of temperature.

- f. The primary jet is correctly and isentropically expanded.

With these assumptions the conservation equations for the balance of mass, momentum and energy at various ejector stations can be written as follows:

**1. Conservation of Mass:**

$$\begin{aligned}\sum(\dot{m})_{p,1,2} &= \dot{m}_3 \\ \sum(\rho A V)_{p,1,2} &= (\rho A V)_3 \\ \sum\left[PAM\sqrt{\frac{W\gamma}{RT_0}\left(1+\frac{\gamma-1}{2}M^2\right)}\right]_{p,1,2} &= \left[PAM\sqrt{\frac{W\gamma}{RT_0}\left(1+\frac{\gamma-1}{2}M^2\right)}\right]_3\end{aligned}\quad (1)$$

**2. Conservation of Momentum:**

$$\begin{aligned}\sum(\dot{m}V+PA)_{p,1,2} &= (\dot{m}V+PA)_3 \\ \sum(\rho AV^2+PA)_{p,1,2} &= (\rho AV^2+PA)_3 \\ \sum[P A(1+\gamma M^2)]_{p,1,2} &= [P A(1+\gamma M^2)]_3\end{aligned}\quad (2)$$

**3. Conservation of Energy:**

$$\begin{aligned}\sum(\dot{m}C_p T_0)_{p,1,2} &= (\dot{m}C_p T_0)_3 \\ \sum\left(\dot{m}\frac{\gamma R}{\gamma-1}T_0\right)_{p,1,2} &= \left(\dot{m}\frac{\gamma R}{\gamma-1}T_0\right)_3 \\ \sum\left(\dot{m}\frac{\gamma}{\gamma-1}\frac{R}{W}T_0\right)_{p,1,2} &= \left(\dot{m}\frac{\gamma}{\gamma-1}\frac{R}{W}T_0\right)_3\end{aligned}\quad (3)$$

Divide the momentum equation (2) by the mass equation (1)

$$\frac{\sum[P A(1+\gamma M^2)]_{p,1,2}}{\sum[PAM\sqrt{\frac{W\gamma}{RT_0}\left(1+\frac{\gamma-1}{2}M^2\right)}]_{p,1,2}} = \frac{[P A(1+\gamma M^2)]_3}{\left[PAM\sqrt{\frac{W\gamma}{RT_0}\left(1+\frac{\gamma-1}{2}M^2\right)}\right]_3}\quad (4)$$

The area ratios  $A_1/A_p$  and  $A_2/A_p$  are related to the mass ratios  $\mu_1 = \dot{m}_1/\dot{m}_p$  and  $\mu_2 = \dot{m}_2/\dot{m}_p$  respectively as follows:

$$\frac{A_1}{A_p} = \mu_1 \frac{P_p M_p}{P_1 M_1} \sqrt{\frac{W_p \gamma_p T_{o1}}{W_1 \gamma_1 T_{op}} \frac{2+(\gamma_p-1) M_p^2}{2+(\gamma_1-1) M_1^2}} \quad (5a)$$

$$\frac{A_2}{A_p} = \mu_2 \frac{P_p M_p}{P_2 M_2} \sqrt{\frac{W_p \gamma_p T_{o2}}{W_2 \gamma_2 T_{op}} \frac{2+(\gamma_p-1) M_p^2}{2+(\gamma_2-1) M_2^2}} \quad (5b)$$

Substitute equation (5a, 5b) in equation (4) and after algebraically gathering terms:

$$\frac{\phi_p + \mu_1 \sqrt{\frac{W_p \gamma_p T_{o1}}{W_1 \gamma_1 T_{op}} \phi_1} + \mu_2 \sqrt{\frac{W_p \gamma_p T_{o2}}{W_2 \gamma_2 T_{op}} \phi_2}}{1 + \mu_1 + \mu_2} = \sqrt{\frac{W_p \gamma_p T_{o3}}{W_3 \gamma_3 T_{op}} \phi_3} \quad (6)$$

where

$$\phi = \frac{1 + \gamma M^2}{M \sqrt{1 + \left(\frac{\gamma - 1}{2}\right) M^2}}$$

Both entrained gases 1 and 2 are the same (atmospheric air) and at the same stagnation pressure and temperature. Squaring both sides of equation (6):

$$\phi_3^2 = \frac{(1 + \gamma_3 M_3^2)^2}{M_3^2 \left(1 + \left(\frac{\gamma_3 - 1}{2}\right) M_3^2\right)} = \left[ \frac{\phi_p + \sqrt{\frac{W_p \gamma_p T_{01}}{W_1 \gamma_1 T_{0p}} \left(\mu_1 \phi_1 + \mu_2 \phi_2\right)}}{1 + \mu_1 + \mu_2} \right]^2 \left( \frac{W_3 \gamma_3 T_{03}}{W_p \gamma_p T_{0p}} \right) \quad (7)$$

This yields the biquadratic equation for the mixed flow Mach number  $M_3$ :

$$\left(\gamma_3^2 - \frac{\gamma_3 - 1}{2} \phi_3^2\right) M_3^4 + (2\gamma_3 - \phi_3^2) M_3^2 + 1 = 0 \quad (8)$$

For  $M_3$  to be real the following condition must be satisfied:

$$\left(2\gamma_3 - \phi_3^2\right)^2 - 4\left(\gamma_3^2 - \frac{\gamma_3 - 1}{2} \phi_3^2\right) \geq 0$$

For the equality condition

$$\phi_3^2 = 2(\gamma_3 + 1)$$

and

$$M_3 = 1$$

or the mixing duct is said to be choked.

With the inequality condition  $M_3$  has two values, one of which is subsonic and the other supersonic.

The detailed computational procedure followed to determine the mixed flow Mach number  $M_3$  and then the ejector thrust for the two stage thrust augmenting ejector is shown in Appendix A.

### 3. RANGE OF CONDITIONS CONSIDERED

#### 3.1 Subsonic/Supersonic Flow Regime

In principle, the available supersonic solutions to the governing equations include the cases  $M_4 > 1$ ,  $M_3 < 1$  and  $M_4 > 1$ ,  $M_3 > 1$ . However, with the assumption of constant area mixing and balanced static pressure across the 1st-2nd stage interface, which were made to limit the potentially enormous number of variables involved, the first of these cases arose only with a narrow range of extremely low first stage area ratios, and the second was impossible. The following results are therefore confined to subsonic flow solutions ( $M_4 < 1$ ,  $M_3 < 1$ ).

#### 3.2 Primary Gas Conditions

Calculations were made in turn with the primary jet consisting of unheated air and hot rocket efflux respectively. Interest in the former case arose both from its relative simplicity and from the fact that many of the experiments in the broader investigation into high pressure ratio thrust augmenting ejectors have been performed with unheated air jets. Jet stagnation pressure was maintained as a variable for the purpose of the calculations, and in all cases the entrained secondary and tertiary flows were of ambient air.

## 4. RESULTS AND DISCUSSION

For a given gas combination,  $P_{op}/P_a$ , and  $T_{op}/(T_{o1}=T_{o2})$  the effectiveness of staging  $\psi = \tau_{2s}/\tau_{1s}$  is a measurement of the improvement in thrust augmentation obtained with the two stage ejector  $\tau_{2s}$  to that with an equivalent single stage ejector  $\tau_{1s}$  having the same area ratio  $A_3/A^*$ . The equivalent single stage ejector, Figure 3b or Figure 3c, is a special case of the two stage ejector, Figure 3a. It can be obtained by fixing the annular area ratio  $A_2/A_1$  at either 0 or  $\infty$ . In this case  $\mu_2 = 0$  or  $\mu_1 = 0$  and the duct of the first stage coincides either with the duct of the second stage or with the nozzle as shown in Figures 3b and 3c respectively.

### 4.1 Air-Air Combination

Typical behaviour at  $\psi$  as a function of  $M_1$  is shown in Figure 4 together with the corresponding variation of  $A_2/A_1$  and  $\Gamma$ . This figure was obtained for fixed values at  $P_{op}/P_a = 42$  and  $A_3/A^* = 800$ . Point A in the figure represents a single stage ejector  $A_2/A_1 = 0$ , and  $\psi = \Gamma = 1$ . In the subsonic range of  $M_1 = 0.299-1.0$ ,  $\psi$  is an increasing function of  $M_1$  from  $\psi = 1$  to a maximum  $\psi = 1.268$  respectively. With further increase in  $M_1$ ,  $\psi$  reduces rapidly to the point where again  $\psi = 1$  at  $M_1 = 1.448$ . Any increase in  $M_1$  beyond this limit will further reduce the two stage ejector performance to be less than that of the equivalent single stage ejector. The supersonic range of  $M_1 = 1.0-1.448$ , at least for stationary conditions which are assumed in these calculations, would require special arrangements such as a sonic throat upstream of the first stage duct as shown in Figure 4. As at present our calculations are for the basic constant cross sectional area ejectors, the supersonic  $M_1$  range will be discarded from the rest of the analysis.

With  $P_{op}/P_a$  fixed at a representative value of 42, and  $T_{op} = T_{o1} = T_{o2}$ , calculations were performed for different area ratios  $A_3/A^*$  with  $A_2/A_1$  and  $M_1$  as parameters. Single stage ejectors in this figure are represented by the curve  $A_2/A_1 = 0$ . It is clear from Figure 5 that for a given  $A_3/A^*$  the maximum thrust augmentation is always obtained at  $M_1 = 1$ . Point A in the figure represents the minimum  $A_3/A^* = 159$  for which a solution is available with  $P_{op}/P_a = 42$ . It will be discussed further in Section 5.3 below. Also for a given  $M_1$ , greater levels of thrust augmentation ratio become available with increasing overall area ratio, i.e. bigger ejectors provide more thrust.

For clearer illustration of the effect of staging, the results of Figure 5 are replotted as  $\psi$  Vs  $A_3/A^*$  in Figure 6a. The single stage ejectors are then represented by the abscissa or  $\psi = 1$ , which is also  $A_2/A_1 = 0$ . The effect of staging on mass augmentation ratio is also shown in Figure 6b.

With a fixed  $A_3/A^* = 200$ , and  $T_{op} = T_{o1} = T_{o2}$ , calculations were performed for different  $P_{op}/P_a$ . The results of  $\psi$  Vs  $M_1$  are shown in Figure 7. In the range  $P_{op}/P_a > 3.9$  the maximum value for  $\psi$  occurs at  $M_1 = 1$ , consistent with previous observations. For  $P_{op}/P_a < 3.9$ ,  $\psi$  (maximum) occurs in the subsonic range of  $M_1$ .

### 4.2 Hot Rocket Gas-Air Combination

The above calculations were repeated but with the hot rocket gas as the primary fluid, entraining atmospheric air at ambient conditions. The properties of this gas are shown in Table I. As a prelude to these calculations the effect of varying the primary fluid temperature was calculated for both single and two stage ejectors, with fixed values of jet stagnation pressure and overall area ratio, using primary gas properties corresponding in turn to air and rocket efflux. The results appear in Figure 8. For two stage ejectors, the calculations were performed only for  $M_1 = 1$ , which corresponds to the maximum  $\tau_{2s}$  obtained by staging. It is clear that thrust augmentation for both single and two stage ejectors deteriorates with increasing primary jet temperature. An important feature of this figure is the limit to  $T_{o1}/T_{op}$  below which no solution can be obtained. At this limit, which is a function of both geometric and primary jet parameters, the flow at the exit of the first mixing duct is sonic,  $M_4 = 1$ , and the mixing duct is said to be choked.

The temperature for the rocket gas used in the analysis is  $2400^\circ K$  or  $T_{o1}/T_{op} = 0.1208$ . This is well below the choking limit  $T_{o1}/T_{op} = 0.245$  shown in Figure 8, for  $P_{op}/P_a = 42$  and

$A_3/A^* = 800$ . From this, and the additional calculations performed with various  $P_{op}/P_a$  and  $A_3/A^*$ , it was found that the inlet Mach number  $M_1$  for which there is a solution is always subsonic for any primary jet stagnation pressure and overall area ratio.

For a fixed  $P_{op}/P_a = 42$ , and  $A_3/A^* = 800$  the results for the H.R.G.-Air,  $T_{o1}/T_{op} = 0.1208$ , and compared with those for Air-Air,  $T_{o1}/T_{op} = 1$ , in Figures 9-11. For the H.R.G.-Air combination,  $M_4$  and  $A_2/A_1$  are rapidly increasing functions of  $M_1$ , and the choking condition occurs at  $M_1 = 0.475$ . For the Air-Air combination  $M_4$  and  $A_2/A_1$  vary with  $M_1$  at much slower rate, and choking of the first stage mixing duct exit does not occur. The mass augmentation ratios are compared in Figure 10, and the effectiveness of staging is shown in Figure 11. Clearly staging with H.R.G.-Air is potentially much less effective than with Air-Air in terms of both thrust and mass augmentation.

For constant  $P_{op}/P_a = 42$ , and different  $A_3/A^*$  the results for both gas combinations are compared in Figure 12, and in Figure 13 for a fixed  $A_3/A^* = 200$  and various  $P_{op}/P_a$ .

In Figures 6 and 7 for Air-Air, and Figures 12 and 13 for H.R.G.-Air the design criteria for a two stage ejector is the  $\psi$  (maximum) which can be obtained for any given  $P_{op}/P_a$ ,  $A_3/A^*$  configurations. These  $\psi$  (maximum) values are plotted in Figure 14 as a function of  $P_{op}/P_a$  and  $A_3/A^*$  for both gas combinations. It is obvious that the effectiveness of staging  $\psi$  increases with the area ratio  $A_3/A^*$  and decreases with increasing pressure ratio  $P_{op}/P_a$ . The effectiveness of staging with the H.R.G.-Air is much less than that with Air-Air gas combination at any  $P_{op}/P_a$  or  $A_3/A^*$ .

#### 4.3 Limiting Conditions

##### 4.3.1 Air-Air Combination

For any constant area ratio  $A_3/A^*$  as shown in Figure 14, the effectiveness of staging decreases with increasing  $P_{op}/P_a$  until a point is reached on the pressure abscissa where  $\psi = 1$  or  $r_{2s} = r_{1s}$ . Any further increase in pressure beyond this limit causes the two stage ejector thrust to be less than that of the equivalent single stage ejector. This limit corresponds to point A in Figures 5 and 6 and in Figure 7, where the pressure curve collapses into a single point A at  $M_1 = 1$ . For a given  $P_{op}/P_a$  and in the range  $A_3/A^* < (A_3/A^*)$  limit the only solutions which can be obtained are mathematical ones involving  $M_1 > 1$  and  $\psi < 1$ . This limiting minimum area ratio can be defined as that at which  $\psi = 1$  and  $M_1 = 1$ , and is shown as a function of  $P_{op}/P_a$  in Figure 15.

##### 4.3.2 Hot Rocket Gas-Air Combination

The limiting condition for the H.R.G.-Air combination is the exit choking condition for the mixing duct of the first stage  $M_4 = 1$ , which was discussed earlier and is also shown in Figure 15.

Only above the limiting curves can a solution be obtained for a two stage ejector with  $\psi \geq 1$ .

### 5. CONCLUSIONS

Calculations based on one dimensional flow theory have been performed for two stage ejectors having constant area mixing ducts. Within the limits imposed by the constraining assumptions—for example, supersonic duct flow solutions could not be fully explored—it is concluded that the two stage ejector is not a viable alternative to the single stage ejector with the high levels of stagnation pressure and temperature pertaining in the efflux of rocket motors. Relative to a single stage ejector with the same overall diameter, staging provides reasonable improvement in thrust augmentation only with prohibitively large diameters.

The one dimensional flow assumptions could take no account of duct length. It is possible that in practice, where the degree of mixing is variable, a two stage ejector of given diameter could reduce the overall length required for a certain level of performance. However, this could be determined only by experiments, which appear barely justifiable on the basis of the above results.

#### **ACKNOWLEDGEMENT**

The author would take this opportunity to express his thanks to Mr S. A. Fisher for his interest in the problem examined in this report, the lengthy discussions, and useful comments.

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## APPENDIX A

### Computational Procedure

For a given  $P_{op}/P_a, T_{op}/(T_{o1} = T_{o2}), A_3/A^*$ , and the properties of the primary and entrained gases, the step by step procedure followed to determine  $M_3$  and then the ejector thrust and thrust augmentation ratio for a two stage ejector was as follows:

$$1. \quad \text{Nominate } M_1, \frac{A_2}{A_1}$$

$$2. \quad \frac{P_{o1}}{P_1} = \left( 1 + \frac{\gamma_1 - 1}{2} M_1^2 \right)^{\frac{\gamma_1}{\gamma_1 - 1}}$$

$$3. \quad \frac{P_{op}}{P_p} = \frac{P_{op}}{P_a} \cdot \left( \frac{P_a}{P_p} = \frac{P_{o1}}{P_1} \right) = \left( 1 + \frac{\gamma_p - 1}{2} M_p^2 \right)^{\frac{\gamma_p}{\gamma_p - 1}}$$

$$\therefore M_p = \left[ \frac{\left( \frac{P_{op}}{P_p} \right)^{\frac{\gamma_p - 1}{\gamma_p}} - 1}{\frac{\gamma_p - 1}{2}} \right]^{1/2}$$

$$4. \quad \frac{A_p}{A^*} = \frac{1}{M_p} \left( \frac{1 + \frac{\gamma_p - 1}{2} M_p^2}{1 + \frac{\gamma_p - 1}{2}} \right)^{\frac{\gamma_p + 1}{2(\gamma_p - 1)}}$$

$$5. \quad \frac{A_4}{A_p} = \frac{\frac{A_3 + A_2}{A_p + A_1}}{1 + \frac{A_2}{A_1}}$$

$$6. \quad \mu_1 = \left( \frac{A_4}{A_p} - 1 \right) \sqrt{\frac{W_1 \gamma_1 T_{op} M_1}{W_p \gamma_p T_{o1} M_p}} \sqrt{\frac{1 + \frac{\gamma_1 - 1}{2} M_1^2}{1 + \frac{\gamma_p - 1}{2} M_p^2}}$$

$$7. \quad \phi_p = \frac{1 + \gamma_p M_p^2}{M_p \sqrt{1 + \frac{\gamma_p - 1}{2} M_p^2}}$$

$$8. \quad \phi_1 = \frac{1 + \gamma_1 M_1^2}{M_1 \sqrt{1 + \frac{\gamma_1 - 1}{2} M_1^2}}$$

$$9. \frac{T_{o4}}{T_{op}} = \frac{1 + \mu_1 \frac{W_p}{W_1} \frac{\gamma_1}{\gamma_p} \frac{\gamma_p - 1}{\gamma_1 - 1} \frac{T_{o1}}{T_{op}}}{1 + \mu_1 \frac{W_p}{W_1} \frac{\gamma_1}{\gamma_p} \frac{\gamma_p - 1}{\gamma_1 - 1}}$$

$$10. \phi_4^2 = \left( \frac{\phi_p + \mu_1 \phi_1 \sqrt{\frac{W_p}{W_1} \frac{\gamma_p}{\gamma_1} \frac{T_{o1}}{T_{op}}}}{1 + \mu_1} \right)^2 \frac{W_4}{W_p} \frac{\gamma_4}{\gamma_p} \frac{T_{op}}{T_{o4}}$$

where

$$W_4 = \left( \frac{\mu_1 + 1}{\mu_1 + \frac{W_1}{W_p}} \right) W_1$$

$$\gamma_4 = \left( \frac{\mu_1 + \frac{\gamma_p}{\gamma_1} \frac{\gamma_1 - 1}{\gamma_p - 1} \frac{W_1}{W_p}}{\mu_1 + \frac{\gamma_1 - 1}{\gamma_p - 1} \frac{W_1}{W_p}} \right) \gamma_1$$

11. Then  $M_4$  which is the mixed flow Mach number at the exit of the first duct is obtained from the following biquadratic equation:

$$\left( \gamma_4^2 - \frac{\gamma_4 - 1}{2} \phi_4^2 \right) M_4^4 + \left( 2\gamma_4 - \phi_4^2 \right) M_4^2 + 1 = 0$$

12. The secondary flow Mach number  $M_2$  at the inlet of the second duct is then calculated using the condition ( $P_2 = P_4$ )

$$\left( 1 + \frac{\gamma_1 - 1}{2} M_2^2 \right)^{\frac{\gamma_1}{\gamma_1 - 1}} = \frac{\mu_1}{1 + \mu_1} \frac{\frac{A_4}{A_p}}{\frac{A_p}{A_1} - 1} \sqrt{\frac{W_4}{W_1} \frac{\gamma_4}{\gamma_1} \sqrt{\frac{T_{o1}}{T_{op}} \frac{T_{op}}{T_{o4}} \frac{M_4}{M_1} \left( 1 + \frac{\gamma_1 - 1}{2} M_1^2 \right)^{\frac{\gamma_1 + 1}{2(\gamma_1 - 1)}}}}}$$

$$\sqrt{1 + \frac{\gamma_4 - 1}{2} M_4^2}$$

$$13. \phi_2 = \frac{1 + \gamma_1 M_2^2}{M_2 \sqrt{1 + \frac{\gamma_1 - 1}{2} M_2^2}}$$

$$14. \mu_2 = \mu_1 \frac{A_2}{A_1} \frac{M_2}{M_1} \left( \frac{\gamma_1 - 1}{1 + \frac{\gamma_1 - 1}{2} M_2^2} \right)^{\frac{\gamma_1 + 1}{2(\gamma_1 - 1)}}$$

$$15. \frac{T_{o3}}{T_{op}} = \frac{1 + \left( \mu_1 + \mu_2 \right) \frac{W_p}{W_1} \frac{\gamma_1}{\gamma_p} \frac{\gamma_p - 1}{\gamma_1 - 1} \frac{T_{o1}}{T_{op}}}{1 + \left( \mu_1 + \mu_2 \right) \frac{W_p}{W_1} \frac{\gamma_1}{\gamma_p} \frac{\gamma_p - 1}{\gamma_1 - 1}}$$

$$16. \quad \phi_3^2 = \left( \frac{\phi_p + (\mu_1 \phi_1 + \mu_2 \phi_2)}{1 + \mu_1 + \mu_2} \sqrt{\frac{W_p}{W_1} \frac{\gamma_p}{\gamma_1} \frac{T_{01}}{T_{op}}} \right)^2 \frac{W_3}{W_p} \frac{\gamma_3}{\gamma_p} \frac{T_{op}}{T_{o3}}$$

where

$$W_3 = \left( \left( \frac{1 + \mu_1 + \mu_2}{1 + \mu_1} \right) \frac{W_2}{W_4} + \mu_2 \right) W_2$$

$$\gamma_3 = \frac{\frac{\mu_2}{1 + \mu_1} + \frac{\gamma_4 \gamma_1 - 1}{\gamma_1 \gamma_4 - 1} \frac{W_1}{W_4}}{1 + \mu_1 + \frac{\mu_2}{1 + \mu_1} + \frac{\gamma_1 - 1}{\gamma_4 - 1} \frac{W_1}{W_4}}$$

17. The final mixed flow Mach number  $M_3$  at the exit of the second duct can be determined from the following biquadratic equation:

$$\left( \gamma_3^2 - \frac{\gamma_3 - 1}{2} \phi_3^2 \right) M_3^2 + \left( 2\gamma_3 - \phi_3^2 \right) M_3^2 + 1 = 0$$

18. For the final mixed flow static pressure at the exit of the second duct to be equal to that of the ambient ( $P_3 = P_a$ ), the following equation must be satisfied:

$$\frac{1 + \mu_1 + \mu_2}{\mu_1} = \sqrt{1 + \frac{\gamma_3 - 1}{2}} M_3^2 \left( 1 + \frac{\gamma_1 - 1}{2} M_1^2 \right)^{\frac{\gamma_1 - 1}{2(\gamma_1 - 1)}} \frac{M_3}{M_1} \frac{\frac{A_3}{A_p}}{\frac{A_4}{A_p} - 1} \sqrt{\frac{W_3 \gamma_3}{W_1 \gamma_1} \sqrt{\frac{T_{01} T_{op}}{T_{op} T_{o3}}}}$$

With the nominated  $M_1$  iterate with  $A_2/A_1$  until this condition is satisfied.

19. The total thrust produced by the (jet-ejector) system exciting to atmospheric pressure is then calculated.

$$F_n + F_e = P_a A_3 \gamma_3 M_3^2$$

20. The Mach number and area of the primary jet when exciting to atmospheric pressure without the ejector is then calculated.

$$\text{MPa} = \left( \left( \frac{P_{op}}{P_a} \right)^{\frac{\gamma_p - 1}{\gamma_p + 1}} - 1 \right)^{1/2}$$

$$\frac{A_{pa}}{A^*} = \frac{1}{\text{MPa}} \left( 1 + \frac{\gamma_p - 1}{2} \text{MPa}^2 \right)^{\frac{\gamma_p + 1}{2(\gamma_p - 1)}}$$

21. The thrust produced by the primary jet when exciting to atmospheric pressure:

$$F_{na} = P_a A_{pa} \gamma_p M_{pa}^2$$

22. The thrust augmentation ratio for a two stage ejector can be obtained.

$$\tau = \frac{F_n + F_e}{F_{na}} = \frac{A_3}{A_{pa}} \frac{\gamma_3}{\gamma_p} \left( \frac{M_3}{M_{pa}} \right)^2$$

TABLE 1

	Rocket Gas	Ambient Air
Ratio of specific heats $\gamma$	1.24	1.4
Molecular Weight $W$	22	29
Primary gas stag. Temp. $T, {}^{\circ}K$	2400	288

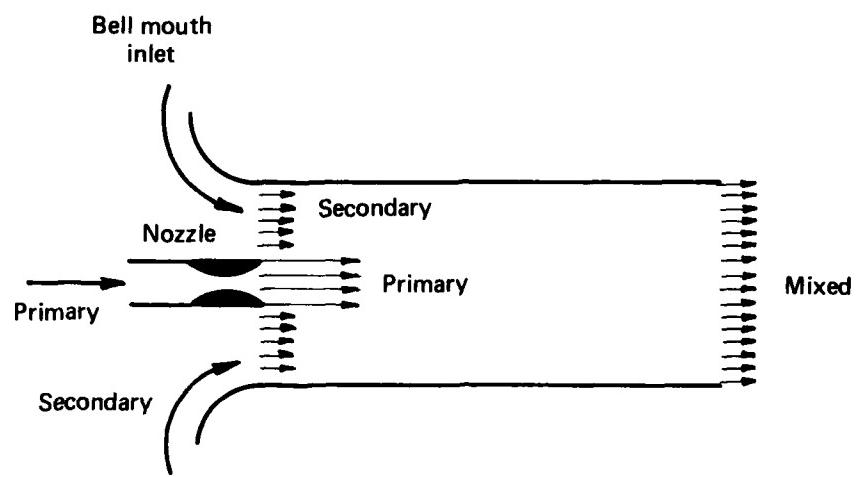


Fig. 1 Schematic diagram for a single stage thrust augmenting ejector.

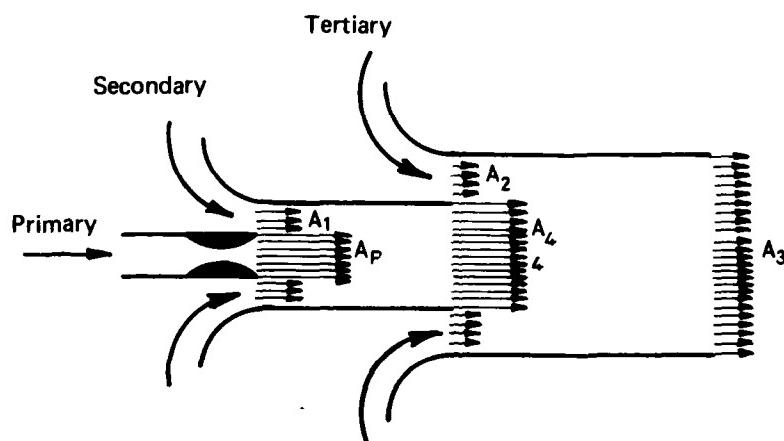
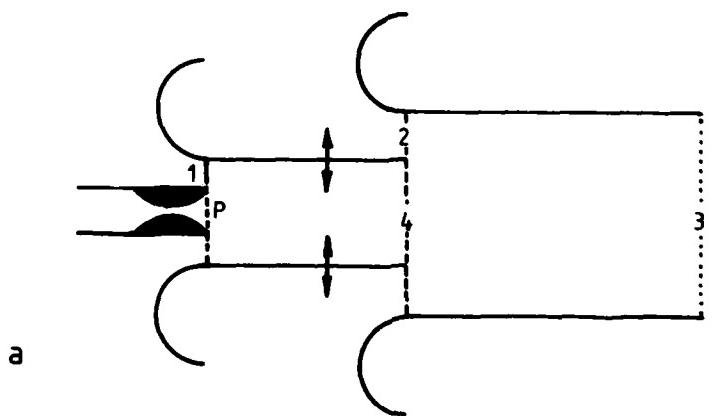
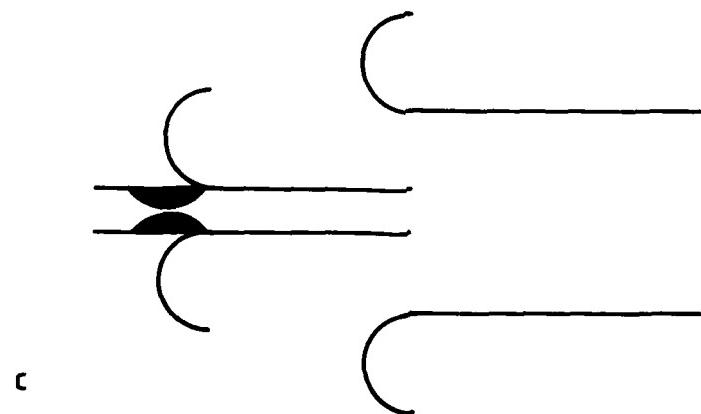
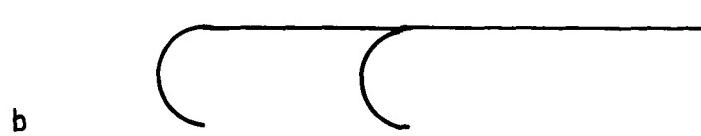


Fig. 2 Schematic diagram for a two stage thrust augmenting ejector.



$$\frac{A_2}{A_1} = 0$$



$$\frac{A_2}{A_1} = \infty$$

Fig. 3 The limiting cases for a two stage ejector to form a single stage ejector.

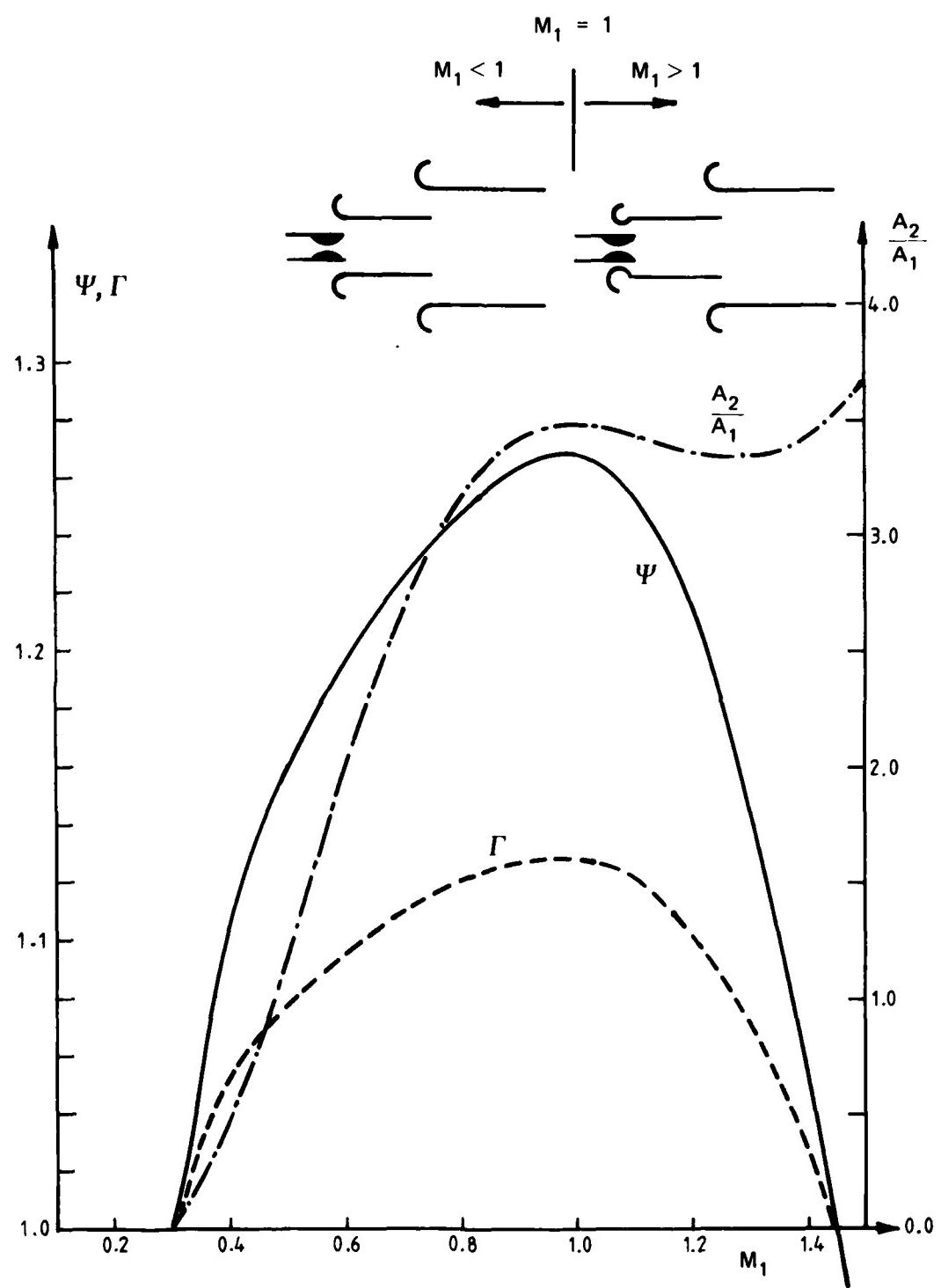


Fig. 4 Performance of two stage thrust augmenting ejector, for  $\frac{P_{op}}{P_a} = 42$ ,  $\frac{A_3}{A^*} = 800$ .

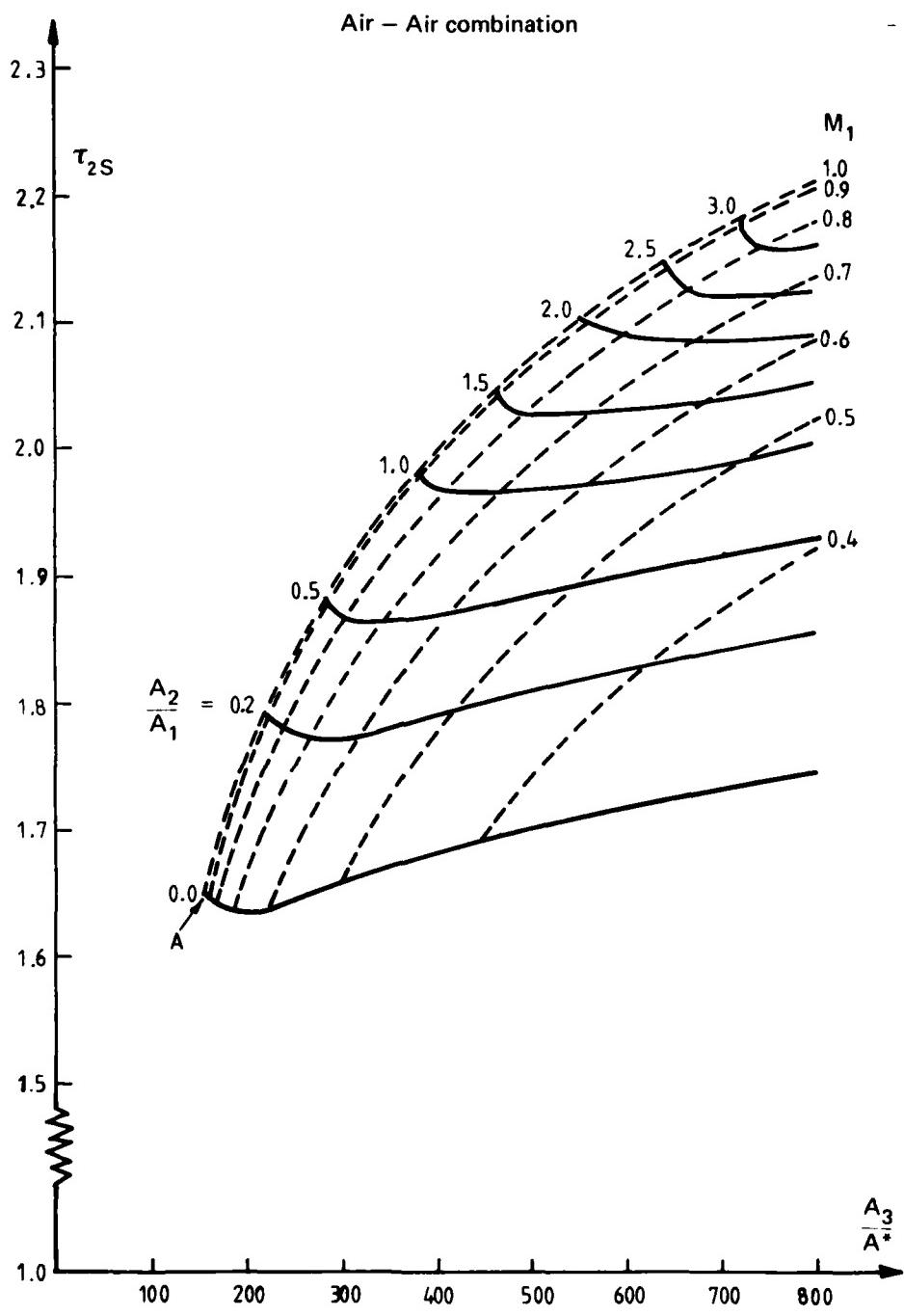
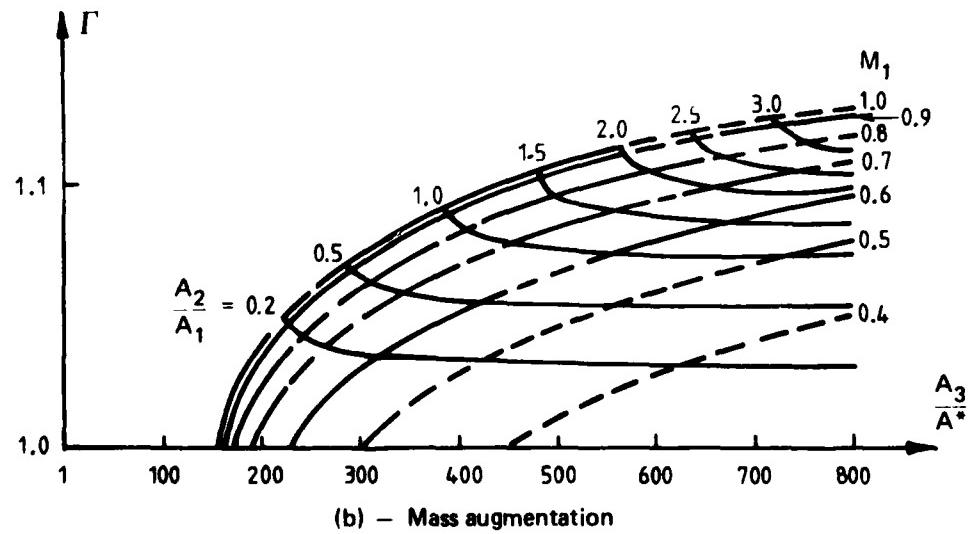
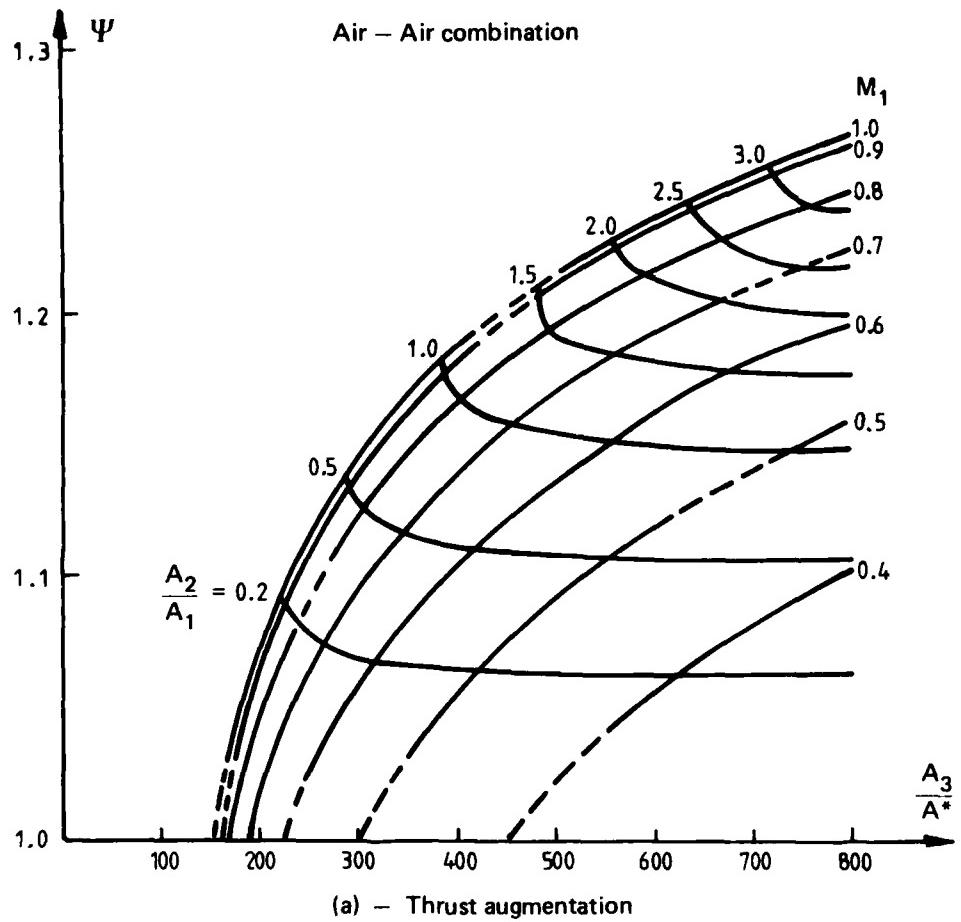


Fig. 5      Performance of two stage thrust augmenting ejectors, for  
 $P_{op} / P_a = 42 = \text{constant}$ ,  $T_{op} = T_{o_1} = T_{o_2}$



**Fig. 6 Performance of two stage thrust augmenting ejectors, for**  
 $\frac{P_{op}}{P_a} = 42 = \text{constant}, T_{op} = T_{o_1} = T_{o_2}$

Air – Air combination

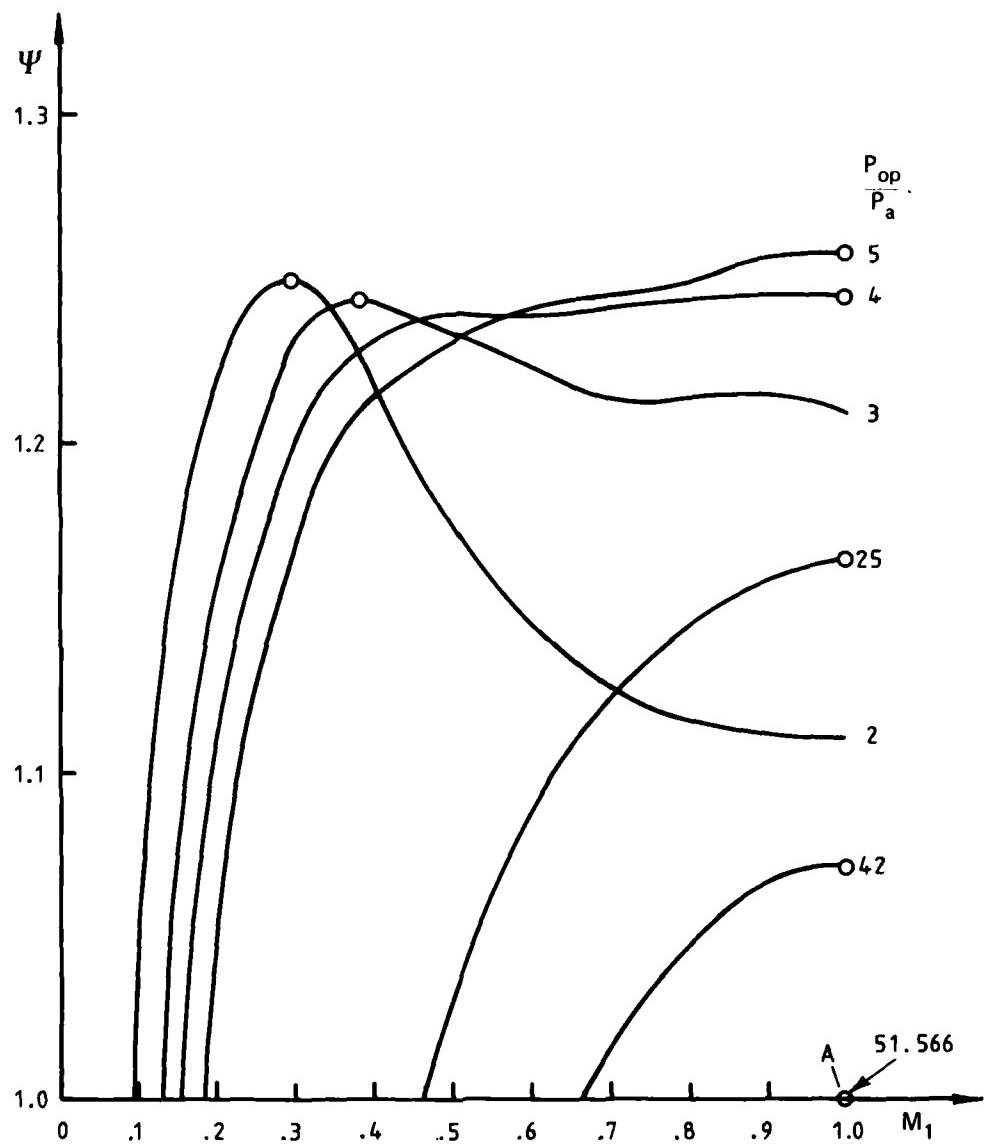


Fig. 7 Results for two stage thrust augmenting ejectors for  $\frac{A_3}{A^*} = 200 = \text{constant}$   
and different  $\frac{P_{op}}{P_a}$ .  $T_{op} = T_{o_1} = T_{o_2}$ ;  $\circ$ ; maximum augmentation.

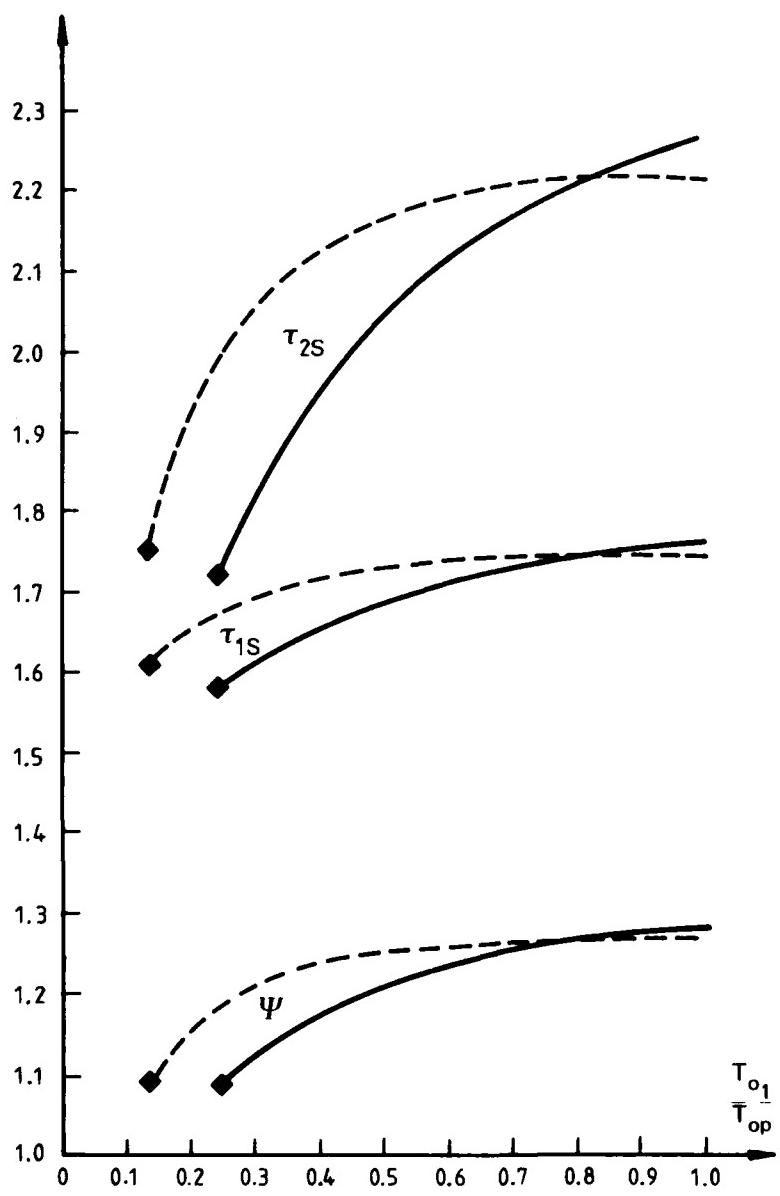
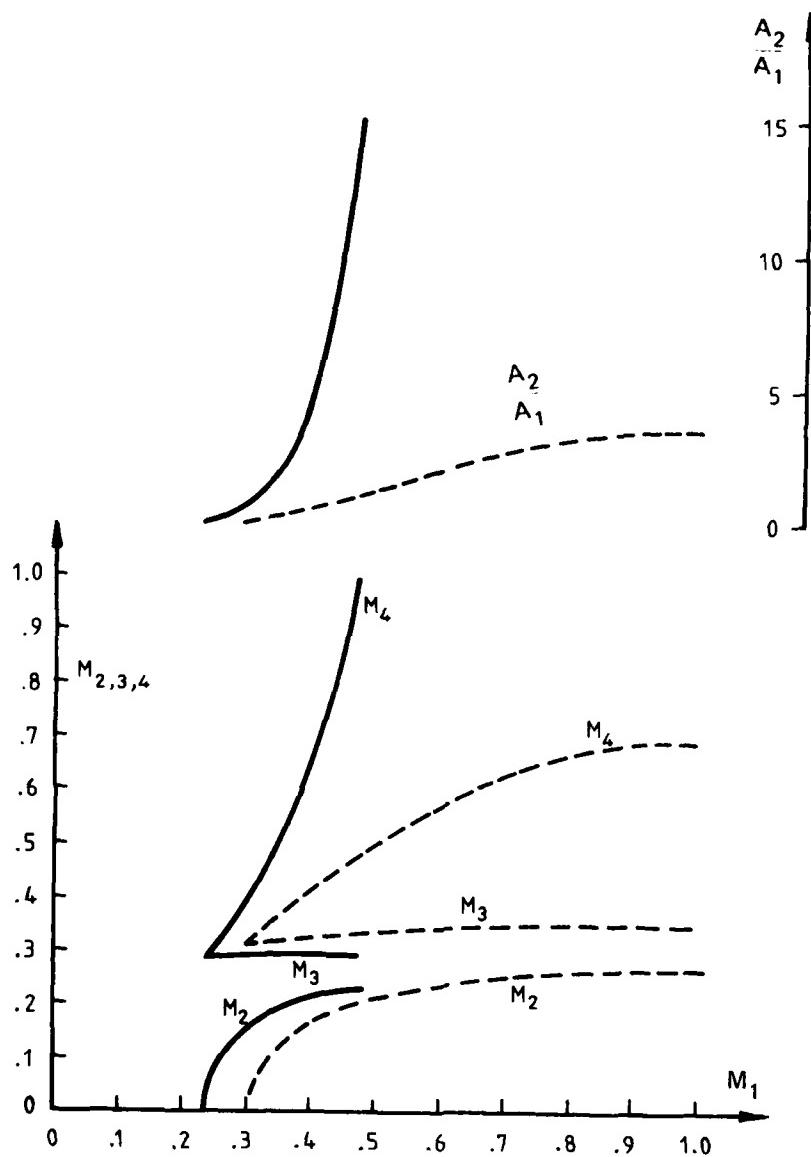


Fig. 8 Effect of temperature of the primary fluid on thrust augmentation for  
 $\frac{P_{op}}{P_a} = 42$ , and  $\frac{A_3}{A^*} = 800$ .  
 $\tau_{2s}$  obtained for  $M_1 = 1.0$ .  
 - - - Air - Air combination, — H.R.G. - Air combination; ◇  
 choking limit ( $M_4 = 1$ ).



**Fig. 9** Comparison of results for two stage thrust augmenting ejectors for both  
 $(\text{--- Air - Air}, T_{op} - T_{o1})$  and  $(-$  H.R.G. - Air,  $\frac{T_{o1}}{T_{op}} = 0.1208$ )

gas combinations.  $\frac{P_{op}}{P_a} = 42$ ,  $\frac{A_3}{A^*} = 800$ .

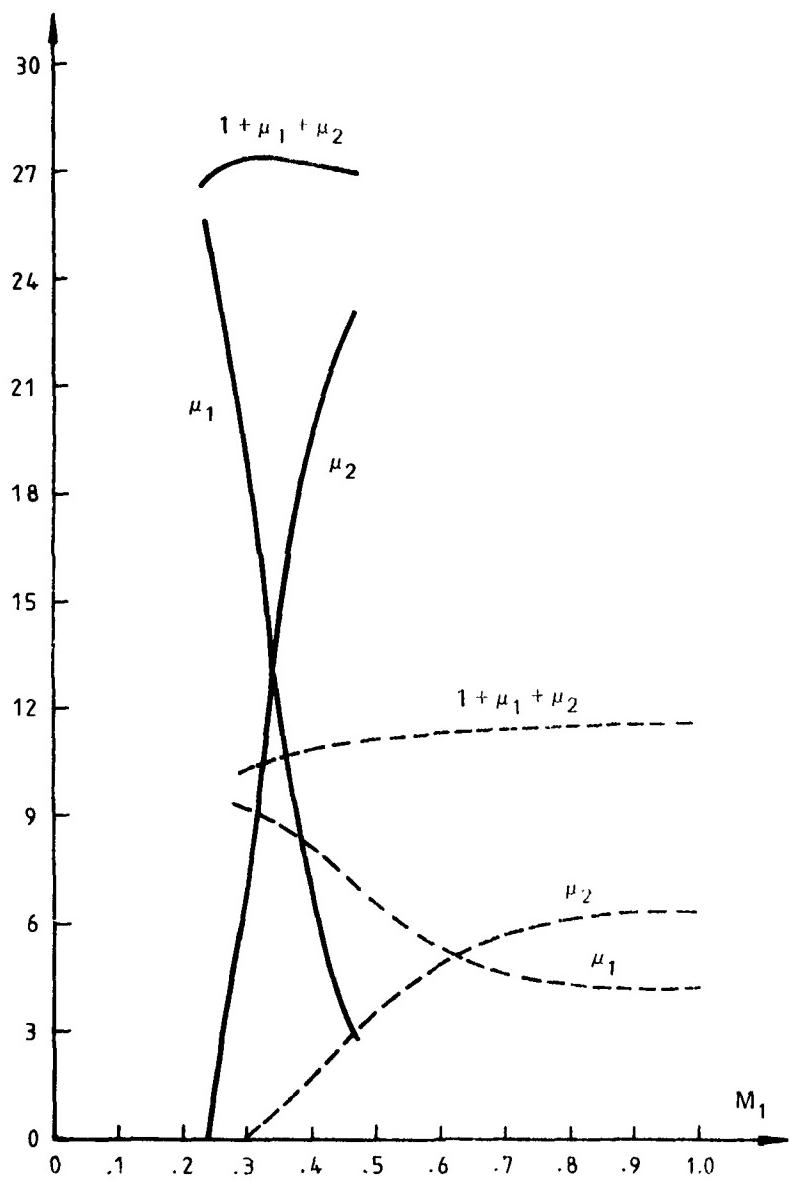


Fig. 10 Comparison of mass augmentation ratios obtained with two stage ejector for both  $\text{--- Air - Air, } T_{op} = T_{o_1}$  and  $\text{— H.R.G. - Air, }$   
 $\frac{T_{o_1}}{T_{op}} = 0.1208$  gas combinations.  $\frac{P_{op}}{P_a} = 42, \frac{A_3}{A^*} = 800.$

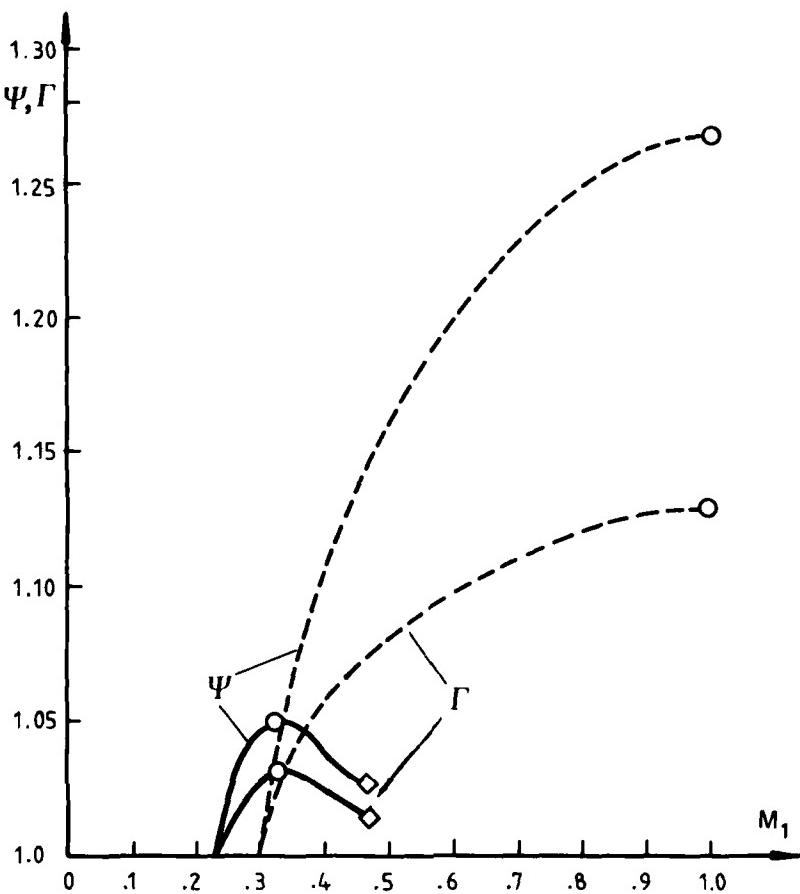


Fig. 11 Comparison of thrust and mass augmentation obtained with two stage ejectors for both - - - Air - Air,  $T_{op} = T_{o_1}$ , and — H.R.G. - Air,  
 $T_{o_1} = 0.1208$  gas combinations  $\frac{P_{op}}{P_a} = 42$ ,  $\frac{A_3}{A_*} = 800$ .  
 ◎ Maximum augmentation, ◇;  $M_4 = 1$

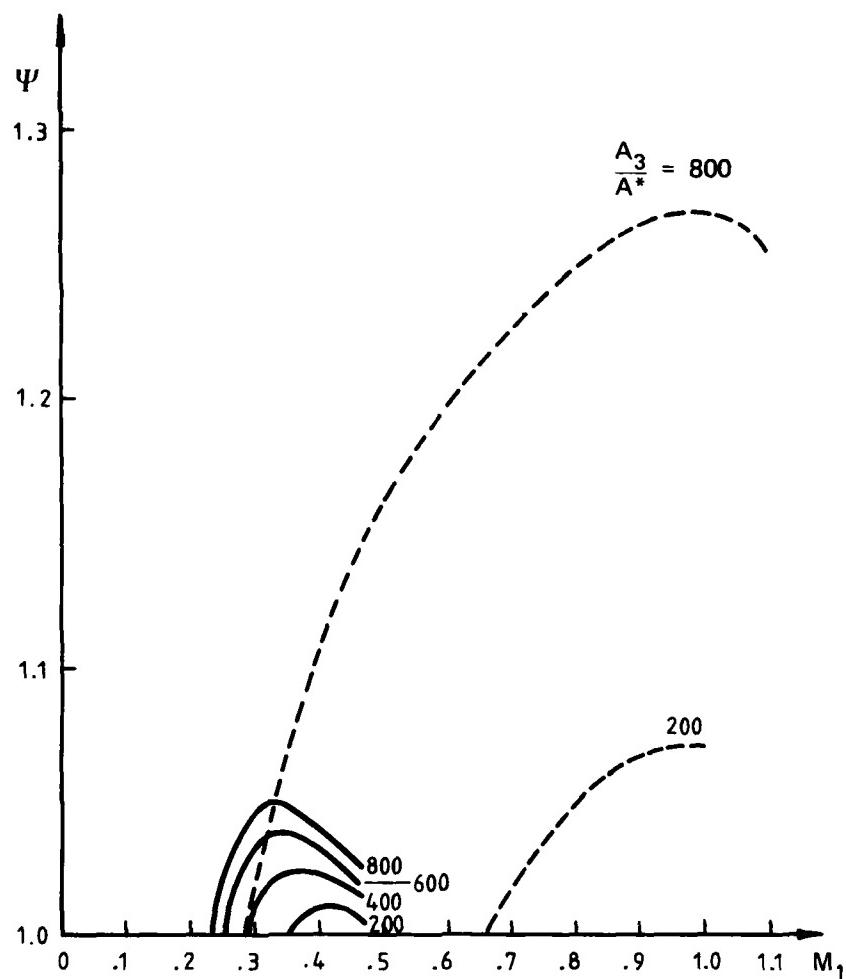


Fig. 12 Comparison of results obtained with two stage thrust augmenting ejectors  
 for both — H.R.G. - Air,  $\frac{T_{02}}{T_{op}} = 0.1208$  and - - - Air - Air,  $T_{op} = T_{01}$   
 gas combinations, for  $\frac{P_{op}}{P_s} = 42 = \text{constant}$ , and variable  $\frac{A_3}{A^*}$

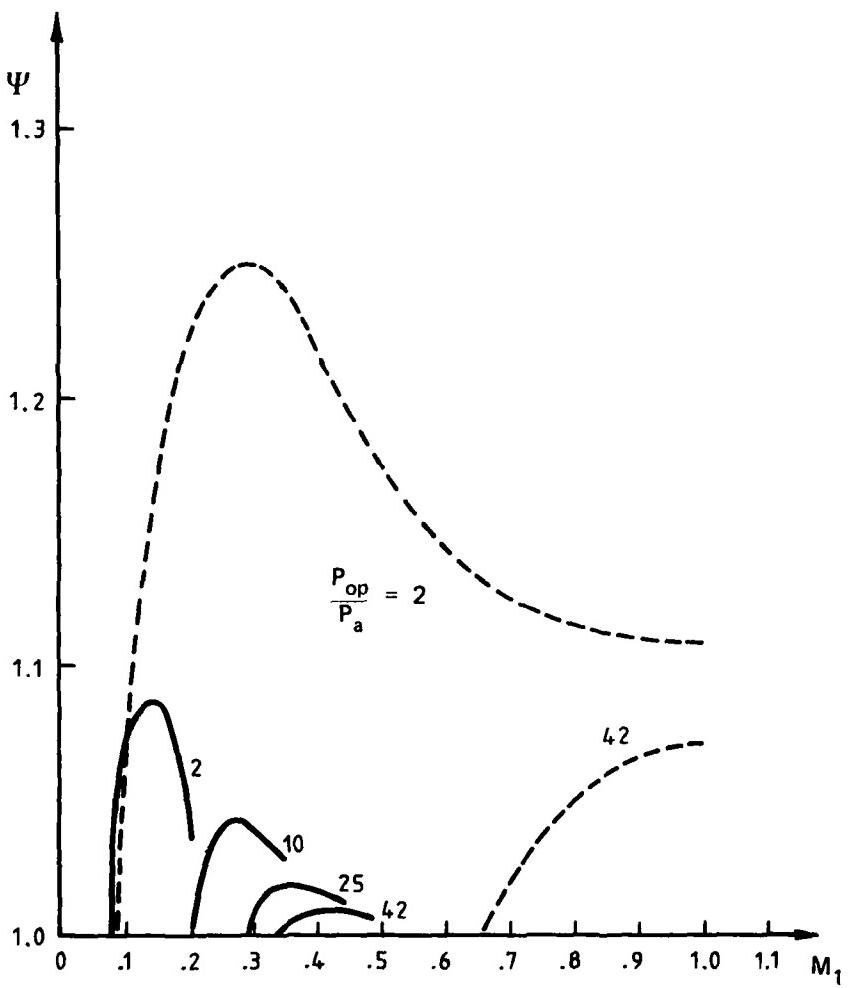


Fig. 13 Comparison of results obtained with two stage thrust augmenting ejectors for  
 $\text{--- Air - Air, } T_{02} = T_{01}$  and  $\text{— H.R.G. - Air, } \frac{T_{02}}{T_{01}} = 0.1208$   
 gas combinations, with  $\frac{A_3}{A_*} = 200 = \text{constant and variable } \frac{P_{02}}{P_{01}}$

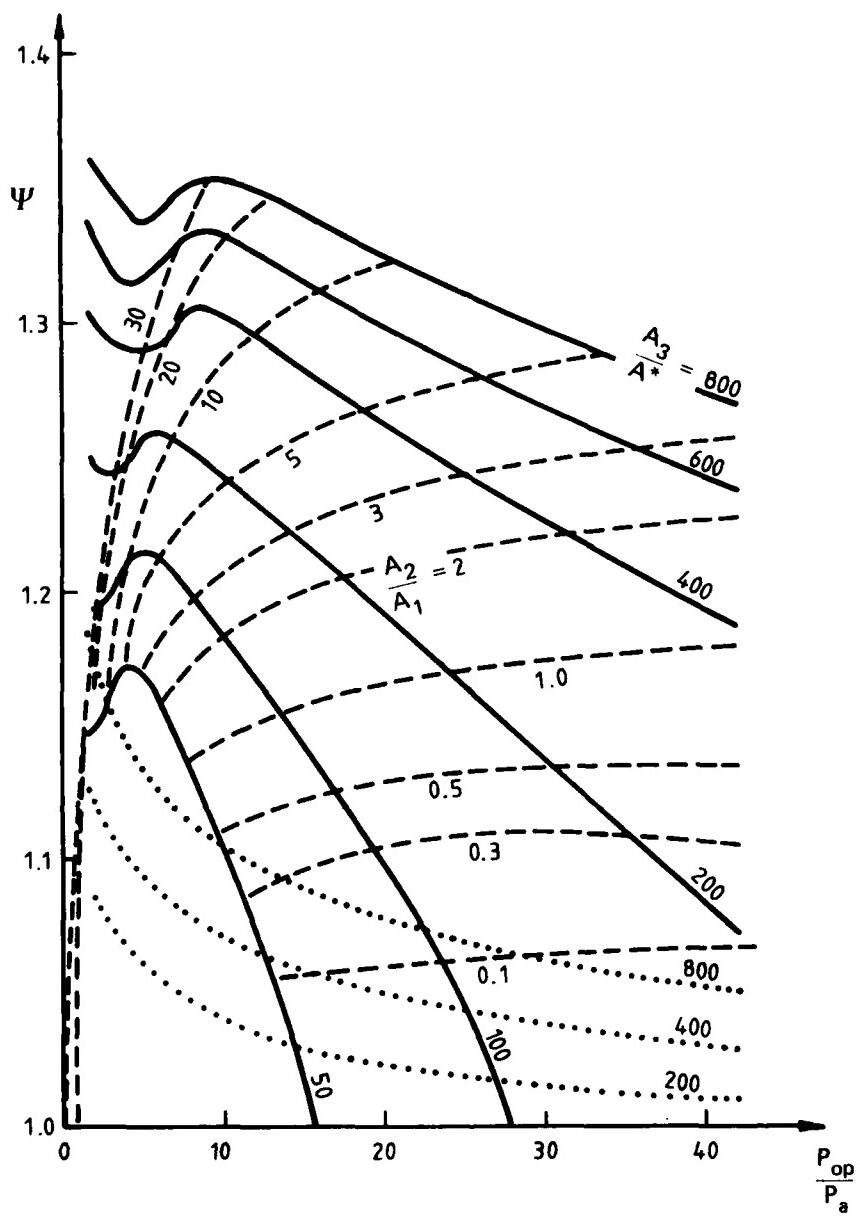
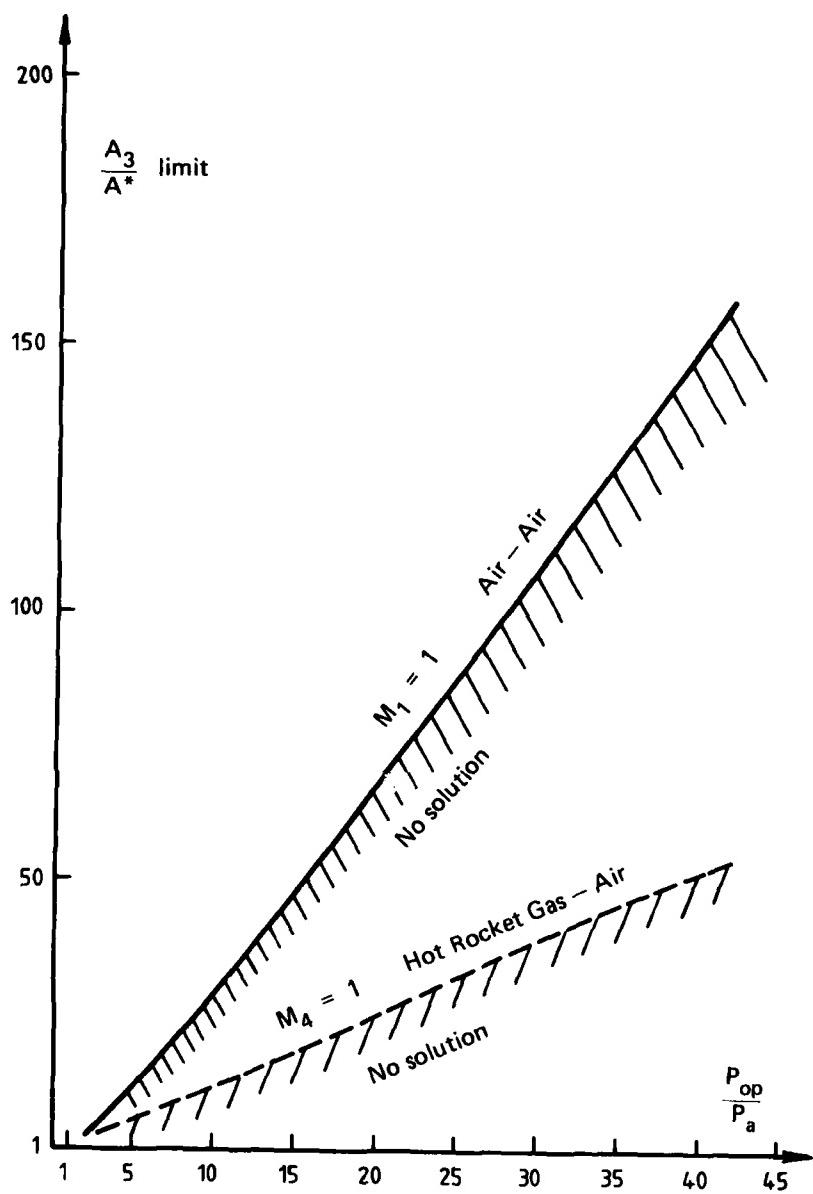


Fig. 14 Results of two stage thrust augmenting ejectors with various  $\frac{P_{op}}{P_a}$  and  $\frac{A_3}{A^*}$  for .... H.R.G. - Air,  $\frac{T_{o1}}{T_{op}}$  = 0.1208 and and — Air - Air,  $T_{o1} = T_{op}$  gas combinations.



**Fig. 15** The limiting conditions for two stage thrust augmenting ejectors.  
 — Air - Air,  $T_{op} = T_{o_1}$  combination for  $\tau_{2s} = \tau_{1s} = 1$  &  $M_1 = 1$ .  
 - - - H.R.G. - Air,  $\frac{T_{o_1}}{T_{op}} = 0.1208$  for the choking limit  $M_4 = 1$ .

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